10.3 Polar Coordinates

Goal: Develop a 2D coordinate system that is good for describing motion/curves that are traveling in a circular/arcing path.

Cartesian Coord.	Polar Coord.
Given (x, y) 1. Stand at origin.	Given (r, θ) 1. Stand at origin facing the positive *x*-axis.
2. Move x-units on x-axis. (positive = right)	2. Rotate by angle θ. (positive = ccw)
3. Move y-units parallel to y-axis. (positive = up)	3. Walk *r*-units in direction you are facing. (neg. = backward)

Example: Plot these points

1. $(r, \theta) = (1, \pi/2)$
2. $(r, \theta) = (3, 5\pi/4)$
3. $(r, \theta) = (0, \pi/3)$
4. $(r, \theta) = (-1, 3\pi/2)$
5. $(r, \theta) = (4, 0)$
6. $(r, \theta) = (4, 100\pi)$

From trig you already know how to convert:

$$x = r\cos(\theta), \qquad y = r\sin(\theta)$$

$$\tan(\theta) = \frac{y}{x}, \qquad x^2 + y^2 = r^2$$

θ	r
0	
π/6	
π/4	
π/3	
π/2	
2π/3	
3π/4	
5π/6	
π	

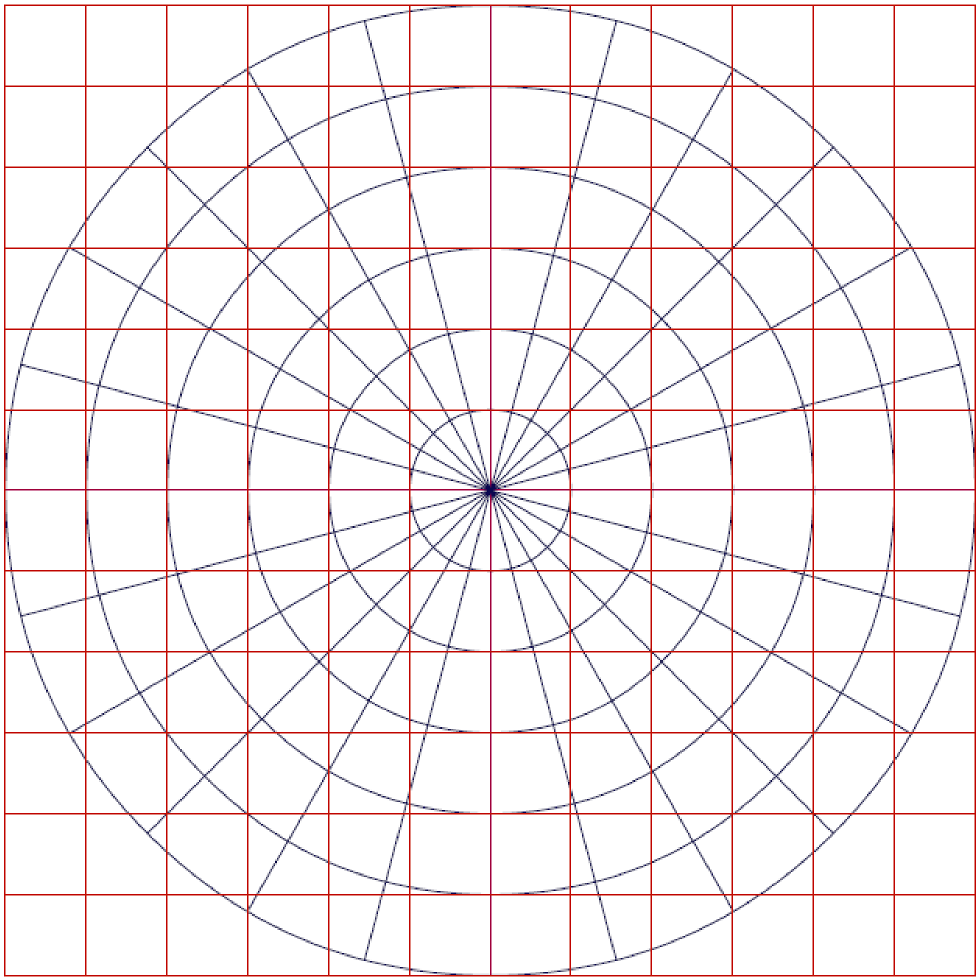
Plotting Polar Curves

1. Can try to convert to x and y.
   Then hope you recognize the curve.
2. Plot points!
   Start with 0, π/2, π, 3π/2.
   For more detail do multiples of
   π/6 and π/4.

Basic Examples:

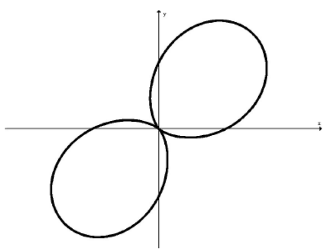
(a) Graph $r = 3$.
(b) Graph $\theta = \pi/4$.
(c) Graph $r = \sin(\theta)$
(d) Graph $r = \cos(2\theta)$

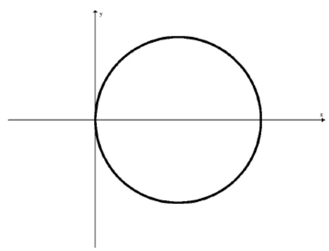
Polar Graph Paper:

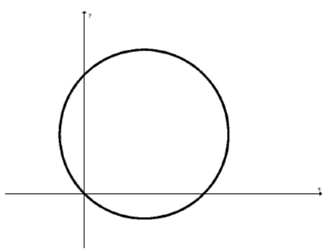


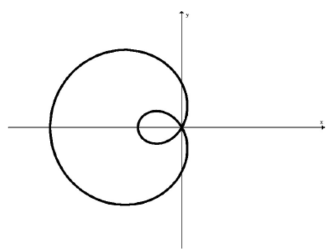
An old exam question:
The four polar equations below each match up with one of the six pictures. Identify which match.

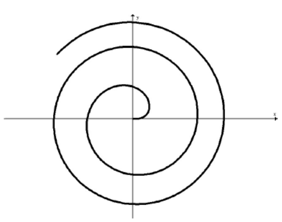
1. $r = \sqrt{\theta}$
2. $r = 1 - 2\cos(\theta)$
3. $r = 1 + \sin(2\theta)$
4. $r = 9\cos(\theta)$

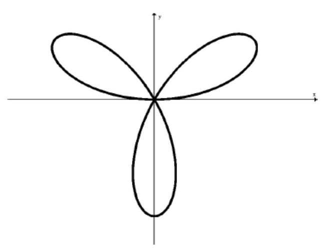
____ 

____ 

____ 

____ 

____ 

____ 

Slopes of tangents for a polar curve

Given a polar curve $r = f(\theta)$.

To find

$\frac{dy}{dx}$ = the slope of the tangent line

here is what we do

1. Note that

$$x = rcos(\theta) = f(\theta)\cos(\theta)$$
$$y = rsin(\theta) = f(\theta)\sin(\theta)$$

2. Use $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f'(\theta)\sin(\theta)+f(\theta)\cos(\theta)}{f'(\theta)\cos(\theta)-f(\theta)\sin(\theta)}$

Since $f'(\theta) = \frac{dr}{d\theta}$, this final answer is often written as

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin(\theta) + r\cos(\theta)}{\frac{dr}{d\theta}\cos(\theta) - r\sin(\theta)}$$